Invariancy of Total Shear Stress for Compressible Turbulent Flows

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IN the present note, a proof of the invariancy of the tota shear stress in free turbulent shear flows is given. It is pointed out that some of the more stringent restrictions imposed by other investigators are not necessary. The Howarth[1] transformation is used to transform the compressible flow field to an incompressible field, and the invariancy of the stream function under this transformation is assumed.

The analysis of compressible turbulent free shear flows is a complex problem. One of the main difficulties in transforming the compressible turbulent flow equations to the incompressible form lies in the transformation of the shear stress term. Some time ago, Mager[2] showed that the Howarth transformation is valid for the compressible turbulent boundary-layer-type problems. However, in applying the transformation he made the assumption that the turbulent shear stresses are invariant under the transformation. Recently Burggraf[3] relaxed the more stringent assumption of Mager by requiring only the y derivative of the turbulent shear to be invariant to permit the use of the transformation. In the present note, the authors show local invariance of the total shear stress across sections perpendicular to the flow direction.

Transformations

The well-known turbulent boundary-layer equations for compressible flows with constant pressure are given by

$$(\partial/\partial x)\bar{\rho}\bar{u} + (\partial/\partial y)\overline{\rho v} = 0 \tag{1}$$

$$\bar{\rho}\bar{u}(\partial\bar{u}/\partial x) + \overline{\rho v}(\partial\bar{u}/\partial y) = (\partial/\partial y)[(\bar{\mu}\partial\bar{u}/\partial y) - \bar{\rho}(\overline{v'u'})] \tag{2}$$

Introducing Howarth's transformation, we get

$$x_* = x \qquad y_* = \int_0^y \frac{\bar{\rho}}{\rho_*}\,dy \tag{3}$$

where ρ_* is a reference density taken to be a constant for a given flow.

Now we define a stream function ψ, which is assumed to be invariant under the previous transformation ($\psi \equiv \psi_*$), as

$$\bar{\rho}\bar{u} = \rho_*(\partial\psi/\partial y) \qquad \overline{\rho v} = -\rho_*(\partial\psi/\partial x) \tag{4}$$

for incompressible flow, or in the transformed plane we have

$$u_* = \partial\psi_*/\partial y_* \qquad v_* = -(\partial\psi_*/\partial x_*) \tag{5}$$

Upon integration of the momentum equation (2), we have

$$\tau = \mu\frac{\partial\bar{u}}{\partial y} - \bar{\rho}(\overline{v'u'}) = \int_0^y \left[\bar{\rho}\bar{u}\frac{\partial\bar{u}}{\partial x} + \overline{\rho v}\frac{\partial\bar{u}}{\partial y}\right] dy + A_0(x) \tag{6}$$

where $A_0(x)$ is a constant of integration. Now applying the transformation on an arbitrary function F we have, from Ref. 3,

$$\bar{\rho}\bar{u}(\partial F/\partial x) + \overline{\rho v}(\partial F/\partial y) = \bar{\rho}[u_*(\partial F/\partial x_*) + v_*(\partial F/\partial y_*)] \tag{7}$$

Hence Eq. (6) in the transformed plane takes the form

$$\tau = \rho_* \int_0^{y_*}\left[u_*\frac{\partial u_*}{\partial x_*} + v_*\frac{\partial u_*}{\partial y_*}\right] dy_* + A_0(x) \tag{8}$$

where the upper limit of integration is the value of y_* corresponding to the upper limit in Eq. (6).

Now integration of the incompressible momentum equation between the same limits of integration as Eq. (8) gives the result

$$\tau_* = \rho_* \int_0^{y_*}\left[u_*\frac{\partial u_*}{\partial x_*} + v_*\frac{\partial u_*}{\partial y_*}\right] dy_* + A_0^*(x_*) \tag{9}$$

Subtracting Eqs. (8) and (9) gives the result

$$\tau - \tau_* = A_0(x) - A_0^*(x_*) = A(x) \tag{10}$$

The total shear stress is thus locally invariant across any axial (x = const) section under the compressibility transformation.

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References

[1] Howarth, L., "Concerning the effect of compressibility on laminar boundary layers and their separation," Proc. Roy. Soc. (London) **A194**, 16 (1948).
[2] Mager, A., "Transformation of the compressible turbulent boundary layer," J. Aeronaut. Sci. **25**, 305–311 (1958).
[3] Burggraf, O. R., "The compressibility transformation and the turbulent-boundary-layer equations," J. Aerospace Sci. **29**, 434–439 (1962).

Effect of Pulse Shape on Final Deformation of Spherical Shells under Impulsive Loading

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Introduction

WHEN an ideal rigid plastic structure is subjected to a load greater than the static collapse pressure, no equilibrium configuration of the stresses can exist, and the structure will accelerate. Obviously, if such a load is applied for any appreciable time, the resulting deformations will become very large, and the structure will no longer be serviceable. However, if the load is applied for an extremely short time, the inertial resistance of the structure may be sufficient to prevent excessive motion, and the usefulness of the structure may not be impaired.

In an earlier investigation[1] the behavior of a spherical shell under stepwise impulsive loading was considered. The present note is concerned with studying the effect of pulse shape on the final deformation of plastic spherical shells. The material of the shell is assumed to be rigid, perfectly plastic, and to obey Tresca's yield condition and the associated flow rule. The load is assumed to be greater than the static collapse pressure and to act for a short period of time.

Specifically, triangular and square waves are considered. It is found that the pulse shape has a profound effect on the final deformation of the shell, even though the peak load and the total impulse of the two waves are adjusted to have the same value. The final deformations of the shell are shown graphically as a function of the peak load.

Basic Equations

The state of stress in a rotationally symmetric shell is described by four generalized stresses: the circumferential and meridional bending moments M_θ and M_ϕ and the circumferential and meridional membrane forces N_θ and N_ϕ.

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